

SELF-SIMILAR SCATTERING OF EVAPORATION PRODUCTS OF A SOLID WALL SUBJECTED TO VARIABLE ENERGY LIBERATION

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A system of self-similar equations of motion of the evaporation products (ideal gas) under the effect of variable energy liberation is considered. Conditions are formulated for the existence and uniqueness of a solution of the problem of evaporation and scattering of a material in a vacuum.

1. In solving the problem of evaporation and scattering of a material in a vacuum with gradual energy liberation, it is ordinarily assumed that the evaporation occurs in a layer of negligibly small thickness, the evaporation products can be considered an ideal gas, and the change in density and velocity of the unevaporated material can be neglected. The motion of the condensed material is therefore not considered, and the problem reduces to considering the motion of an ideal, variable-mass, gas layer bounded by the evaporation surface on one side and a free surface on the other. Such a schematization of the phenomenon is used in [1-3], for example.

The formation of the evaporation conditions is essential in formulating the problem in the approximation mentioned. These conditions connect the parameters of the evaporated and unevaporated material on both sides of the evaporation surface and govern the law of its motion. Since the motion of the unevaporated material and the structure of the layer where evaporation occurs are not considered, then the evaporation conditions are not known beforehand and are formulated from some physical reasoning about the process as a whole. These conditions should assure the existence and uniqueness of the solution of the problem.

The existence and uniqueness of the solution of the problem about the scattering of the surface evaporation products from a solid wall in a vacuum are considered herein in a specific example of self-similar motion.

2. At the initial instant $t=0$ the condensed material occupies the half-space $x \geq 0$ and is characterized by a density ρ_0 , a pressure p_0 , and a zero velocity.

The material adjoins the vacuum at the point $x=0$. For $t > 0$ energy is liberated in the material at the rate

$$\partial E / \partial t = Q = C m^{-\alpha} t^{2-1} \quad (2.1)$$
$$\left(m = \int_0^x \rho dx \right)$$

where E is the specific internal energy, m is the Lagrange mass coordinate, C and α are constants, where $C > 0$ and

$$0 < \alpha < 1 \quad (2.2)$$

Condition (2.2) means that a finite energy is liberated in a finite mass between 0 and x in the finite time from 0 to t . Moreover, for $0 < \alpha < 1$ the rate of energy liberation diminishes as m and t grow.

The motion of vapor subjected to the energy liberation (2.1) is described by a system of gasdynamics equations

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$$\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{\partial p}{\partial m}, & \frac{\partial V}{\partial t} &= \frac{\partial u}{\partial m} \\ \frac{\partial E}{\partial t} &= -p \frac{\partial V}{\partial t} + Q, & E &= \frac{pV}{\gamma-1} \end{aligned} \quad (2.3)$$

where u is the velocity, V is the specific volume, γ is the Poisson adiabatic index. The boundary condition on the free surface is $p=0$ for $m=0$. It is assumed that the evaporation conditions do not depend on parameters with independent dimensionalities to the already existing governing parameters of the problem $\rho_0, p_0, C, \alpha, \gamma$.

The motion with the governing parameters is self-similar, with a self-similarity index of one. Let us limit ourselves to evaporation conditions for which the evaporation surface, which is a plane, will move deep in the surface at a constant velocity.

3. Let us introduce the dimensionless variables

$$\lambda = \frac{m}{u_0 \rho_0 t}, \quad U = \frac{u}{u_0}, \quad P = \frac{p}{\rho_0 u_0^2}, \quad v = V \rho_0 \quad (2.3)$$

$$(u_0 = (C / \rho_0^\alpha)^{1/(2+\alpha)})$$

The system (2.3), written in the variables (3.1), becomes

$$\begin{aligned} v' &= q \lambda^{-(1+\alpha)} \Delta^{-1}, & P' &= -q \lambda^{1-\alpha} \Delta^{-1}, & U' &= -q \lambda^{-\alpha} \Delta^{-1} \\ \Delta &= \lambda^2 v - \gamma P & (q &= (\gamma-1) \rho_0^{(4+\alpha)/(2+\alpha)} C^{4/(2+\alpha)}) \end{aligned} \quad (3.2)$$

The first two equations of the system (3.2) can be examined independently of the third. The velocity can be found by a quadrature with an additive constant for $v(\lambda)$ and $P(\lambda)$ known. The system (3.2) has been investigated partially in [3].

Let us introduce the new variables s, l, z by means of the formulas

$$s = \lambda^2, \quad l = \gamma P / s v, \quad z = (\lambda^{2+\alpha} v^2)^{-1} \quad (3.3)$$

The first two equations of the system (3.2) become in the new variables

$$\begin{aligned} \frac{dz}{ds} &= \frac{z [2qz - (2+\alpha)(l-1)]}{2s(l-1)} \\ \frac{dl}{ds} &= \frac{qz(l+\gamma) - 2l(l-1)}{2s(l-1)} \end{aligned} \quad (3.4)$$

Equating the numerators of the right sides of (3.4) to zero, we obtain two particular solutions

$$\begin{aligned} l &\equiv 0, \quad z \equiv 0 \\ l = l_1 &= \gamma \frac{2+\alpha}{2-\alpha}, \quad z = z_1 = \frac{2+\alpha}{2q} (l_1 - 1) \end{aligned} \quad (3.5)$$

The first solution corresponds to $\rho \equiv 0$ and is of no interest.

The second solution, together with the velocity equation, results in

$$v = z_1^{-1/2} \lambda^{(2+\alpha)/2}, \quad P = \frac{2+\alpha}{2-\alpha} \lambda^{(2-\alpha)/2} z_1^{-1/2}, \quad U = -\frac{2+\alpha}{\alpha} \lambda^{-\alpha/2} + U_0 \quad (3.6)$$

Here U_0 is an arbitrary constant. The particular solution (3.6) has been mentioned in [3]. Dividing the first equation in (3.4) by the second, we obtain

$$\frac{dz}{dl} = \frac{z [2qz - (2+\alpha)(l-1)]}{qz(l+\gamma) - 2l(l-1)} \quad (3.7)$$

Therefore, the problem of integrating the system (3.2) reduces to the integration of (3.7) since for a known dependence $z(l)$ the solution of the remaining equations reduces to quadratures.

In order to analyze the existence and uniqueness of the solution of the problem about vapor scattering in a vacuum, let us examine the field of integral curves of (3.7). The integral curves are shown schematically in Fig. 1. The arrows indicate the direction of growth in the variable s .

Two integral curves enter the singularity A (saddle point) with the coordinates $l = l_1, z = z_1$.

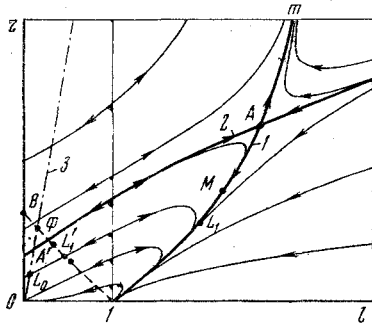


Fig. 1

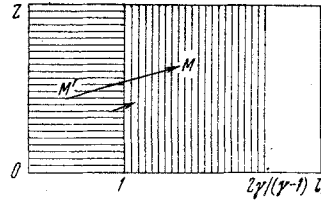


Fig. 2

It follows from the second equation in (3.4) that the relationship for s

$$l = \psi |s|^y + l_1$$

is valid in the neighborhood of the point A, where ψ is an arbitrary constant. Let us note that $y > 0$ along the integral curve 1 (Fig. 1) for all admissible values of α , q , and γ . Along this integral curve $s = \lambda^2 \rightarrow 0$ as $l \rightarrow l_1$. Along the second integral curve A $y < 0$ at the point A, hence $s \rightarrow \pm\infty$ as $l \rightarrow l_1$. It follows from (3.3) that in the neighborhood of the point A

$$P = \frac{l_1}{\gamma \sqrt{z_1}} s^{(2-\alpha)/4}, \quad v = \frac{1}{\sqrt{z_1}} s^{-(2+\alpha)/4} \quad (3.8)$$

It is seen from (3.8) that $P \rightarrow 0$, $v \rightarrow \infty$ upon approaching the point A along the first curve, i.e., in this case the point A corresponds to the boundary with the vacuum.

The singularity $l = 0$, $z = 0$ is a node. As an analysis of the system (3.4) shows, the variable $s \rightarrow +\infty$ upon approaching the point O from both directions. As the singularity $l = 1$, $z = 0$ is approached, the variable $s \rightarrow \text{const} \neq 0$.

The infinitely remote point $l = \infty$, $z = \infty$ is also singular. The integral curves enter it from the direction $dz/dl = \alpha/q$. Hence, the variable $s \rightarrow 0$ according to the law $s = Hl^{-2/(2-\alpha)}$, where H is an arbitrary constant. The equations

$$P \rightarrow P_1 = \frac{1}{\gamma} \sqrt{\frac{q}{zH}}, \quad \rho = \rho_0 \sqrt{\frac{\alpha}{q} H} s^{\alpha/2} \rightarrow 0, \\ U = \frac{q}{\gamma(1-\alpha)P_1} s^{(1-\alpha)/2} + U_0$$

are valid for the density, pressure, and velocity in the neighborhood of the point $l = \infty$, $z = \infty$, where U_0 is an arbitrary constant. Therefore, the point $l = \infty$, $z = \infty$ corresponds to the boundary with a piston of constant nonzero pressure.

In the neighborhood of any nonsingular point (l_0, z_0)

$$\frac{dl}{ds} = \frac{qz(l, \gamma) - 2l_0(l, -1)}{2(l_0 - 1)s} = \frac{\Phi}{s} \neq 0$$

from which

$$s = Ne^{l\Phi}$$

Here N is a nonzero arbitrary constant, i.e., $s \neq 0$ at nonsingular points of (3.7). Thus, the point A is the single point which corresponds to the boundary with the vacuum, hence the boundary condition for (3.7), corresponding to the boundary with the vacuum, is $z = z_1$ for $l = l_1$ along curve 1 (Fig. 1).

4. Since the vapor flow may be discontinuous (with a shock), let us consider the relation between l and z on both sides of the discontinuous self-similar solutions. From the Hugoniot relationships

$$V_+ - V_- = -W^{-1}(u_+ - u_-), \quad u_+ - u_- = W^{-1}(p_+ - p_-) \\ E_+ - E_- = 1/2(p_+ + p_-)(V_- - V_+)$$

the relationship between $(l, z)_-$ and $(l, z)_+$ follows in the form

$$z_+ = z_- \left(\frac{\gamma + 1}{\gamma - 1 + 2l_-} \right)^2, \quad l_+ = \frac{2\gamma - (\gamma - 1)l_-}{\gamma - 1 + 2l_-} \quad (4.1)$$

Here W is the shock velocity, the minus subscript refers to the state ahead of the front, and the plus to the state behind the front. It follows from (4.1) that the points of the line $l=1$ go over into themselves, i.e., $l=1$ corresponds to a weak discontinuity. This also follows from Eq. (3.3) for l , which can be re-written as $l = (c\rho/\lambda)^2$ since the speed of sound is $c = \sqrt{\gamma pV}$ in an ideal gas.

The points with $l_- < 1$ transform into points with $l_+ > 1$; hence, $z_+ > z_-$, meaning $\rho_+ > \rho_-$ also, which corresponds to a compression shock. Points with $l_- > 1$ transform into points with $l_+ < 1$; hence, $z_+ < z_-$, which corresponds to a rarefaction shock. Rarefaction shocks are mechanically unstable in an ideal gas; hence, the points with $l > 1$ cannot correspond to the states ahead of the shock front. The possible shock passages are shown in Fig. 2. The domain of possible states ahead of the front are hatched horizontally, and behind the front, vertically. The line $l=0$ goes over into the line $l = 2\gamma/(\gamma-1)$.

The dashed line 1B in Fig. 1 is the locus of states ahead of the shock front which go over into points of the curve 1.

5. The information presented about the integral curves of (3.4) and (3.7), as well as about the properties of the self-similar shocks, is adequate for going over to a consideration of the existence and uniqueness of the solution of the problem about vapor scattering in a vacuum. The solution of (3.7) which satisfies the condition $z = z_1$ at $l = l_1$ along curve 1 exists and is unique if the boundary condition in the plane of evaporation is mapped in the l, z variables by:

- 1) a point belonging to curve 1;
- 2) a line having just one point of intersection with curve 1 but no common points with triangle 1 0 B;
- 3) a point belonging to the triangle 1 0 B.

There is no solution or it is not unique in the remaining cases.

Indeed, in the first case the solution is continuous and consists of pieces of the separatrix MA, where M is a point corresponding to the surface of evaporation. If this point coincides with A, then the problem is solved by using the particular solution (3.6).

The second case reduces to the first, but the initial values of l and z are not given in advance, but are determined by the intersection between the separatrix 1 and a line corresponding to the boundary condition.

In the third case the solution is discontinuous and consists of a piece of the integral curve L_0L_1' and a piece of the curve L_1A , where L_0 is the point corresponding to the evaporation surface and L_1' is the intersection between the integral curve and the line 1B, while L_1 is the point into which the point L_1' transforms on the shock (Fig. 1).

Let us examine the case of spoilage of the existence and uniqueness conditions of the solution to (3.7). If a point corresponding to the surface of evaporation does not belong to the triangle 1 0 B and does not lie on the separatrix 1, then it is seen from Fig. 1 that there is no integral curve (continuous or discontinuous) which would connect this point to the point A while retaining the monotonicity of the change in the variable s . (The solution in which the variable $s = \lambda^2$ does not change monotonically has no physical meaning.) If a line belonging to the triangle 1 0 B corresponds to the boundary condition on the evaporation surface, then an infinite set of discontinuous solutions exists, starting at points of this line. The solutions are constructed exactly as in the third case.

To construct a unique solution of (3.4) for known values of l and z on the evaporation surface, it is necessary and sufficient to assign the velocity D of the evaporation surface. This follows from (3.4). To find the dependence $U(\lambda)$ it is necessary and sufficient to assign the vapor velocity u_1 on the evaporation surface. This follows from the third equation in the system (3.2).

It follows from the structure of the solution to (3.7) that two qualitatively distinct modes of self-similar vapor scattering are possible:

- 1) continuous flow;
- 2) flow with a shock.

The solution of the problem of vapor scattering in a vacuum exists and is unique if the velocity of the evaporation surface and the velocity of the vapors thereon as well as the density and pressure or their relationship can be determined from the evaporation conditions.

The following evaporation conditions have been formulated in [3] for energy liberation of the form (2.1). The law of evaporation surface motion is given by the equation

$$\int_0^t Q(m_1, t) dt = c_v(T_1 - T_0) + \theta = \beta \quad (5.1)$$

from which it follows that the surface velocity is $D = (\alpha\beta)^{-1/2} C^1/\alpha$.

Here m_1 is a coordinate of the evaporation surface, T_0 and C_v are the initial temperature and specific heat of the condensed material, and T_1 and θ are the temperature and specific heat of evaporation. According to (5.1) the evaporation surface at time t is at a point where the temperature has been raised because of energy liberation, from the initial to the evaporation temperature, and energy equal to the heat of evaporation has been liberated. The remaining evaporation conditions are

$$(u_1 - D)\rho_1 = -D\rho_0, \quad p_1 = RT_1\rho_1 \quad (5.2)$$

where ρ_1 , p_1 , and u_1 are the vapor density, pressure, and velocity, and R is the universal gas constant. In the l, z variables, these conditions yield the line

$$z = l(\tau\lambda_1^\alpha)^{-1} \quad (\tau = RT_1/u_0^2, \lambda_1 = D/u_0) \quad (5.3)$$

The line (5.3) passes through the origin; hence (3.7) has an infinity of discontinuous solutions which start at points in the interval 0Φ of the line (5.3) (Fig. 1). Equations (3.4) and (3.2) also have an infinite number of solutions in this case; hence the evaporation conditions (5.1) and (5.2) should be acknowledged unsuitable.

The example considered for self-similar nonadiabatic motion shows that the physical consistency of the evaporation conditions still does not ensure the existence and uniqueness of the solution of the problem. An investigation of the evaporation conditions from this viewpoint is necessary.

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